**ENGINEERING MECHANICS**

Engineering Mechanics is the study of bodies which are in motion or at rest. Branch of mechanics that deals with bodies at rest is called **STATICS**. Branch of mechanics that deals with moving bodies is called **DYNAMICS**.

![Diagram](https://via.placeholder.com/150)

If the analysis involves the force acting on the body, it is called **kinetics**. If analysis does not involve force causing the motion, it is called **kinematics**.

**SCALAR QUANTITIES**

These are parameters having only magnitude and no direction. ex: time, distance, speed, mass, energy.

**VECTOR QUANTITIES**

These are parameters having both magnitude and direction. ex: velocity, acceleration, torque.
NEWTON'S FIRST LAW OF MOTION

Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external unbalanced force.

NEWTON'S SECOND LAW OF MOTION

The rate of change of momentum of a body is directly proportional to the applied force on the body. And the motion takes place in the direction of force.

\[ F \propto \frac{dp}{dt} \]
\[ \propto \frac{dmv}{dt} \]
\[ \propto m\frac{dv}{dt} \]
\[ \propto ma \]
\[ F = ma \]

NEWTON'S THIRD LAW OF MOTION

Every action has an equal and opposite reaction.

EQUATIONS OF MOTION

1) \[ v = u + at \]
2) \[ s = ut + \frac{1}{2}at^2 \]
3) \[ v^2 = u^2 + 2as \]

Where \( v \rightarrow \) final velocity (m/s), \( u \rightarrow \) initial velocity (m/s), \( a \rightarrow \) acceleration (m/s\(^2\)), \( t \rightarrow \) time (s) & \( s \rightarrow \) displacement (m)
1. Newton's First Law of Motion.
6. Principle of Transmissibility of Forces.

**Newton's Law of Gravitation**

It states that every body is attracted by every other body with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

\[ F = \frac{GM_1M_2}{r^2} \]

**Parallelogram Law of Vector Addition**

If two vectors can be represented in magnitude and direction as two adjacent sides of a parallelogram then the resultant of these two vectors can be represented as the
diagonal of parallelogram in magnitude and direction.

Resultant \( R \) is given by,

\[
R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}
\]

where \( \alpha \) is the angle between \( P \) and \( Q \).

**PRINCIPLE OF TRANSMISSIBILITY OF FORCES.**

A force acting on any point on the body can be moved to any other point along the line of action of the force without producing any change in the external effect on the body.
LAMI'S THEOREM

It states that when three forces acting on a point are in equilibrium, then magnitude of each force is directly proportional to sine of angle between other two forces.

By Lami's theorem,

\[ \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \]

? A body of weight \( 50 \text{N} \) is hanging on two strings \( AB \) and \( BC \) as shown in Fig. Determine the tension in each AB and BC.
At point B.

By Lami's theorem,

\[
\frac{H}{\sin 110^\circ} = \frac{T_{AB}}{\sin 130^\circ} = \frac{T_{BC}}{\sin 120^\circ}
\]

\[
\frac{50}{\sin 110^\circ} = \frac{T_{AB}}{\sin 130^\circ} = \frac{T_{BC}}{\sin 120^\circ}
\]

\[
T_{AB} = \frac{50}{\sin 110^\circ} \times \sin 130^\circ = 40.76 \text{ N.}
\]

\[
T_{BC} = \frac{50}{\sin 110^\circ} \times \sin 120^\circ = 46 \text{ N}
\]

? An electric bulb of 20 kg is hanging as shown in Figure. Determine the tension in the strings.

Point Q.
By Lami's theorem.
\[ \frac{W}{\sin 70^\circ} = \frac{TQR}{\sin 130^\circ} = \frac{TPQ}{\sin 160^\circ}. \]

\[ TQR = \frac{20 \times 9.81 \times \sin 130^\circ}{\sin 70^\circ} = 159.94 \, N \]

\[ TPQ = \frac{20 \times 9.81 \times \sin 160^\circ}{\sin 70^\circ} = 71.4 \, N \]

A ball of weight 30 N is hanging on a cable as shown in Figure. It is supported in a vertical wall. Determine the tension in the cable and reaction by the wall.

![Diagram of forces acting on the center.]

By Lami's theorem
\[ \frac{W}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin 150^\circ}. \]

\[ T = \frac{30}{\sin 120^\circ} \times \sin 90^\circ = 34.64 \, N \]

1) An electric bulb of mass \(5\) kg is hanging as shown in figure. Determine the tension acting on the cable.

\[
R = \frac{80 \times \sin 150^\circ}{\sin 120^\circ} = 17.32\ N
\]

2) A ball of mass \(12\) kg and radius \(100\) mm is hanging as shown in figure. Determine the tension on the cable and reaction force on the ball. Given length of cable is \(150\) mm.

1) A At the centre.
By Lami's theorem,

\[
\frac{W}{\sin 70^\circ} = \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 140^\circ}
\]

\[
T_1 = \frac{5 \times 9.81 \times \sin 150^\circ}{\sin 70^\circ} = 26.09 \text{ N}
\]

\[
T_2 = \frac{5 \times 9.81 \times \sin 140^\circ}{\sin 70^\circ} = 33.55 \text{ N}
\]

2) A

\[
\sin \theta = \frac{100}{150} = 0.66.
\]

\[
\theta = 41.81^\circ.
\]

By Lami's theorem,

\[
\frac{W}{\sin 131.8^\circ} = \frac{R}{\sin 138.2^\circ} = \frac{T}{\sin 90^\circ}
\]

\[
R = \frac{12 \times 9.81 \times \sin 138.2^\circ}{\sin 131.8^\circ} = 105.25 \text{ N}
\]

\[
T = \frac{12 \times 9.81 \times \sin 90^\circ}{\sin 131.8^\circ} = 157.91 \text{ N}
\]
RESULTANT OF TWO FORCES

Two forces $P$ and $Q$ are acting on a point as shown in figure. Angle btw $P$ & $Q$ is $\theta$.

By completing the parallelogram we have as the resultant of $P \& Q$.

Let $\theta$ is the angle than $R$ makes with $P$.

From $\triangle OCB$,
\[ \tan \theta = \frac{BC}{OB} \]  \hspace{1cm} (1)

Consider $\triangle ABC$
\[ \sin \alpha = \frac{BC}{AC} = \frac{BC}{Q} \]
\[ BC = Q \sin \alpha \]
\[ \cos \alpha = \frac{AB}{AC} = \frac{AB}{Q} \]
\[ AB = Q \cos \alpha \]

$\Rightarrow \tan \theta = \frac{BC}{OA + AB}$
\[ = \frac{Q \sin \alpha}{P + Q \cos \alpha} \]

$\theta = \tan^{-1} \left( \frac{Q \sin \alpha}{P + Q \cos \alpha} \right)$
Two forces $P$ and $Q$ of magnitude 12 N and 10 N make an angle of $50^\circ$. Blowing them, determine the magnitude and direction of resultant.

$P = 12 \text{ N}$
$Q = 10 \text{ N}$
$\alpha = 50^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$= \sqrt{12^2 + 10^2 + 2 \times 12 \times 10 \times \cos 50^\circ}$$

$$= \sqrt{144 + 100 + 2 \times 120 \cos 50^\circ}$$

$$= 19.96 \text{ N}$$

$$\theta = \tan^{-1} \left( \frac{Q \sin \alpha}{P + Q \cos \alpha} \right)$$

$$= \tan^{-1} \left( \frac{10 \sin 50^\circ}{12 + 10 \cos 50^\circ} \right)$$

$$= 21.29^\circ$$

**RESULTANT OF SEVERAL FORCES**

Consider a system of forces as shown in the figure. Let $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ are the angles with positive $x$-axis. Then,
Then resultant force in \( \alpha \)-direction can be written as

\[
\sum F_x = P_1 \cos \theta_1 + Q \cos \theta_2 + R \cos \theta_3 + S \cos \theta_4 + T \cos \theta_5
\]

\[
\sum F_y = P \sin \theta_1 + Q \sin \theta_2 + R \sin \theta_3 + S \sin \theta_4 + T \sin \theta_5
\]

Resultant,

\[
R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}
\]

\[
\theta = \tan^{-1} \left( \frac{\sum F_y}{\sum F_x} \right)
\]

Five forces are acting on a point as shown in figure. Determine the resultant of system of forces.

\[
\begin{align*}
\theta_1 &= 0 \\
\theta_2 &= 30^\circ \\
\theta_3 &= 180^\circ \\
\theta_4 &= 220^\circ \\
\theta_5 &= 340^\circ \\
\sum F_x &= P_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + F_4 \cos \theta_4 + F_5 \cos \theta_5 \\
&= 10 \cos 0 + 20(\cos 30^\circ) + 15(\cos 180^\circ) + 25(\cos 220^\circ) + 5(\cos 340^\circ) \\
&= -21.3
\end{align*}
\]
\[ \Sigma F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + F_4 \sin \theta_4 + F_5 \sin \theta_5 \]

\[ = 10 \sin 0^\circ + 20 \sin 30^\circ + 15 \sin 180^\circ + 25 \sin 220^\circ + 5 \sin 348^\circ \]

\[ = -7.79 \]

\[ R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \]

\[ = \sqrt{(-2.13)^2 + (-7.79)^2} \]

\[ = 8.05 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{\Sigma F_x}{\Sigma F_y} \right) = \tan^{-1} \left( \frac{-7.79}{-2.13} \right) \]

\[ = 74.67^\circ \]

? A system of forces are acting along a regular hexagon as shown in figure. Determine the resultant of system of forces.

\[ \begin{align*}
A: & \\
\theta_1 &= 0^\circ & \theta_4 &= 90^\circ \\
\theta_2 &= 30^\circ & \theta_5 &= 120^\circ \\
\theta_3 &= 60^\circ & 
\end{align*} \]
\[ \Sigma F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + F_4 \cos \theta_4 + F_5 \cos \theta_5 \]
\[ = 20 \cos 0 + 10 \cos 30 + 5 \cos 60 + 10 \cos 90 + 5 \cos 120 \]
\[ = 23.66 \text{ N} \]

\[ \Sigma F_y = 20 \sin 0 + 10 \sin 30 + 5 \sin 60 + 10 \sin 90 + 15 \sin 120 \]
\[ = 32.32 \text{ N} \]

\[ R = \sqrt{\left( \Sigma F_x \right)^2 + \left( \Sigma F_y \right)^2} \]
\[ = \sqrt{(23.66)^2 + (32.32)^2} = 40.05 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1} \left( \frac{32.32}{23.66} \right) \]
\[ = 53.79^\circ \]

**MOMENT DUE TO A FORCE**

Moment due to a force is given by

\[ m = F \times r \]

where \( F \) is the magnitude of force

and \( r \) is the distance from the point to the line of action of force.

Unit of moment is Nm.

Moment due to a force can be either clockwise or anticlockwise.
In the fig. four forces $f_1, f_2, f_3, f_4$ are acting in different directions. Moment at O,
due to $f_1 = (f_1 \times l_1)$
due to $f_2 = (f_2 \times l_2)$
due to $f_3 = (f_3 \times l_3 \times \sin \theta_3)$
due to $f_4 = (f_4 \times l_4 \times \sin \theta_4)$

Generally moment in clockwise direction is taken as negative and moment in anticlockwise direction is taken as positive.

Determine the total moment at point A due to the system of forces shown in the fig.

![Diagram](image)

1. Moment at A,
due to $30N = (30 \times 1) \text{Nm} \uparrow = 30 \text{Nm} \uparrow$
due to $10N = \left(\frac{10 \times 2 \times \sin 30^\circ}{2} \right) \text{Nm} \downarrow = -10 \text{Nm} \downarrow$
due to $20N = (20 \times 4 \times \sin 50^\circ) \text{Nm} \uparrow = 61.28 \text{Nm} \uparrow$
due to $15N = (15 \times 5 \times \sin 20^\circ) \text{Nm} \downarrow = -25.65 \text{Nm} \downarrow$

Total moment at A = $30 - 10 + 61.28 - 25.65$

$= 55.63 \text{Nm} \uparrow$ (since +ve)
Principle of moments states that the moment due to the resultant of a system of non-concurrent co-planar forces is equal to the algebraic sum of moments due to individual forces in the system about the same point.

Replace the system of four forces in the given system by a single force.

Resultant of the given system of forces, \( R \)
\[
R = -10 + 20 - 5 + 10 = -5 \text{ N}
\]

Moment due to given system of forces at \( A \):
\[
= (-10 \times 2) + (20 \times 3) + (-5 \times 4) + (-10 \times 6)
= -20 + 60 - 20 - 60 = -40 \text{ Nm}
\]

Let the resultant is at a distance of \( x \) m from point \( A \) as shown in the fig.

By principle of moments,
\[-(R \times \alpha) = -40\]

\[-5 \alpha = -40\]

\[\alpha = \frac{-40}{-5} = 8 \text{ m}\]

\[\alpha = 8 \text{ m}\]

Replace the system of forces shown in Fig. by a single force.

A Resultant of the given system of forces,

\[R = +20 - 5 + 10 - 15 = 10 \text{ kN}\]

Moment due to given system of forces at O

\[= (20 \times 2) + (-5 \times 4) + (10 \times 6) + (-15 \times 7)\]

\[= 40 - 20 + 60 - 105\]

\[= -25 \text{ kNm}\]

By principle of moments,

\[+ (R \alpha) = -25\]

\[+ 10 \alpha = -25\]

\[\alpha = \frac{-25}{10} = -2.5 \text{ m}\]
Negative sign indicates the resultant force should be replaced to the left of support.

Replace the system of forces by a force and moment at point A.

![Diagram of forces](image)

- Resultant of system of forces:
  \[ R = -10 - 15 + 20 = -5 \text{ N} \]
  
  Moment due to given system of forces,
  \[ = (10 \times 2) + (-15 \times 4) + (20 \times 5) \]
  \[ = -20 - 60 + 100 = 20 \text{ Nm} \]

The given system of coplanar forces consists of four forces. Determine the equivalent by using

a) a single force.

b) force and couple at point A.
c) Force and couple at point B.

\[ \begin{align*}
10 \text{ kN} & \quad R \quad 5 \text{ kN} \\
& \quad 3 \text{ kN} \quad 2 \text{ kN}
\end{align*} \]

\[ \begin{array}{c}
A \quad 2 \text{ m} \quad 2 \text{ m} \quad 2 \text{ m} \quad 1 \text{ m} \quad 1 \text{ m} \\
B
\end{array} \]

\[ -10 + 5 + 3 - 2 = -4 \text{ kN} \]

Moment due to given system of forces,
\[ = (-10 \times 2) + (5 \times 4) + (3 \times 6) + (2 \times 7) \]
\[ = -20 + 20 + 18 - 14 = 4 \text{ kNm} \]

\[ -R \alpha = 4 \]
\[ -4 \alpha = 4 \]
\[ \alpha = -1 \text{ m} \]

The force should be on left side of point A.

b)

Moment due to counter-clockwise
\[ 4 \text{ kNm} \]

\[ R = -4 \text{ kN} \]

Moment of these forces at B
\[ = (10 \times 6) - (5 \times 4) - (3 \times 2) + (2 \times 1) \]
\[ = 60 - 20 - 6 + 2 \]
\[ = 36 \text{ kNm} \]
The system of four forces as shown in Fig is to be replaced by an equivalent system. Find equivalent system.

a) Through point A
b) Through point B
c) By a single force

A a) Resultant of the system of forces,

\[ R = -3 - 2 + 4 + 5 = 4 \text{ N} \]

Moment due to given system of forces,

\[ = (-3 \times 1) + (-2 \times 2) + (4 \times 3) + (5 \times 4) \]

\[ = -3 - 4 + 12 + 20 = 25 \text{ Nm} \]
b) \( R = 4 \text{ N} \)

Moment of these forces at point B:

\[
\begin{align*}
&= (3 \times 4) + (2 \times 3) + (4 \times 2) + (5 \times 1) \\
&= -12 - 6 + 8 + 5 = +5 \text{ Nm}
\end{align*}
\]

\[ \text{4 N.} \]

\[ \text{5 Nm.} \]

---

c) \[ R \alpha = 25 \]

\[ 4 \alpha = 25 \]

\[ \alpha = \frac{25}{4} = 6.25 \text{ m} \]

FREE BODY DIAGRAM

Free body is the representation of all the forces acting on the body, with its magnitude and direction.

Step 1: Isolate the body from its surroundings.

Step 2: Represent all the forces acting on the body in magnitude and direction.

ex:

\[ \text{FBD} \]

\[ \text{FBD} \]
EQUILIBRIUM OF BODIES

Equilibrium of bodies subjected to two forces is shown in figure. If the body is in equilibrium, \( F_2 \) must be equal to \( F_1 \) and opposite to \( F_1 \).

Equilibrium of bodies subjected to three forces. When a body subjected to three forces is in equilibrium, all the forces must pass through a single point.

In the figure, \( F_1, F_2, F_3 \) are three forces acting on the body. Let \( R \) is resultant of \( F_1 \) and \( F_2 \), \( R \) must be equal to \( F_3 \) and opposite to \( F_3 \), if the body is to be in equilibrium. Here \( R \) can be called as resultant force and \( F_3 \) can be called as equilibrant.
Equilibrant is the force which when added with resultant produces zero.

Conditions for Equilibrium in 3-D.

If a body is at rest it must satisfy the following conditions.

\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
\[ \sum F_z = 0 \]
\[ \sum m = 0 \]

Writing in component form:

\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
\[ \sum F_z = 0 \]
\[ \sum m = 0 \]

These are equilibrium conditions in 3-D.

A string which is rigid is connected between point A and B as shown in figure. If a mass of 10 kg is connected to point C. Determine the mass connected on point D to make the system in eqm. Assume portion CD of the string is parallel to the plane.
A join CE and DF

Assume \( AE = x \).

\( FB = 5 - x \)

we have \( CE = DF = y \) (parallel)

Consider \( \Delta AEC \),

\[
\begin{align*}
  y^2 &= 3^2 - x^2 \\
  y^2 &= 9 - x^2 \quad (1)
\end{align*}
\]

Consider \( \Delta BDF \),

\[
\begin{align*}
  y^2 &= 4^2 - (5 - x)^2 \\
  &= 16 - 25 + 10x - x^2 \\
  y^2 &= 10x - x^2 - 9 \quad (2)
\end{align*}
\]

Equating (1) \& (2)

\[
9 - x^2 = 10x - x^2 - 9
\]

\[
10x = 18
\]

\[
\frac{18}{10} = 1.8 \text{ m}
\]

\[
\theta_1 = \cos^{-1} \left( \frac{1.8}{3} \right) = 33.13^\circ
\]

\( BF = 3.2 \)

\[
\theta_2 = \cos^{-1} \left( \frac{3.2}{4} \right) = 36.87^\circ
\]

Apply eqn condition at point C

Apply eqn condition at point C
\[ \Sigma F_x = 0 \]
\[ \Rightarrow T_{CD} - T_{CA} \cos 53.1 = 0 \tag{3} \]

\[ \Sigma F_y = 0 \]
\[ \Rightarrow T_{CA} \sin 53.1 - W = 0 \tag{4} \]
\[ T_{CA} \sin 53.1 = 98.1 \]
\[ \Rightarrow T_{CA} = 122.67 \text{ N} \]
\[ T_{CA} \text{ in equation (3)} \]

\[ \Rightarrow T_{CD} = 98.6 \text{ N.} \]

At point D

\[ \Sigma F_x = 0 \]
\[ \Rightarrow T_{BD} \cos 36.86 - T_{CD} = 0 \tag{5} \]

\[ \Sigma F_y = 0 \]
\[ \Rightarrow T_{BD} \sin 36.86 - H = 0 \tag{6} \]
\[ T_{BD} \cos 36.86 - 93.6 = 0 \]
\[ (5) \Rightarrow T_{BD} = 94.9 \approx 92 \text{ N.} \]

\[ (6) \Rightarrow 92 \sin 36.86 - H = 0 \]
\[ H = 55.1 \text{ N} \]

\[ M = \frac{H}{9} = \frac{55.1}{9.81} = 5.6 \text{ kg} \]
A ball of weight 300N is resting on a plane as shown in Figure. Determine the reaction at all contact points.

\[
\begin{align*}
\text{FBD} & \\
\text{NORTH will be up to plane, and dir inward.}
\end{align*}
\]

Applying \( \sum F_x = 0 \),

\[
R_A \cos 60^\circ - R_B \cos 50^\circ = 0
\]

\[
R_A = R_B \cos 50^\circ.
\]

\[
\frac{R_A}{2} = 1.28 R_B
\]

\[\sum F_y = 0\]

\[
R_A \sin 60^\circ + R_B \sin 75^\circ - 300 = 0.
\]

\[
R_B \frac{\sqrt{3}}{2} + R_B \sin 75^\circ - 300 = 0.
\]

\[
R_B = 160.43 \text{ N}
\]
A ball of mass 20 kg is resting on a plane as shown in Fig. 1. Determine the reactions.

A Free Body Diagram (FBD)

Applying eqm. conditions:

\[ \sum F_x = 0 \]
\[ R_p \cos 70^\circ - R_q \cos 75^\circ = 0 \]
\[ R_p = \frac{\cos 75^\circ}{\cos 70^\circ} R_q \]
\[ R_p = 0.76 R_q \]

\[ \sum F_y = 0 \]
\[ R_p \sin 70^\circ + R_q \sin 75^\circ = 196.2 \]
\[ 0.76 R_q \sin 70^\circ + R_q \sin 75^\circ = 196.2 \]
\[ 0.71 R_q + 0.97 R_q = 196.2 \]
\[ R_q = \frac{196.2}{1.68} = 116.78 \text{ N} \]
\[ R_p = 0.76 R_q = 88.95 \text{ N} \]
Two balls of equal size and weight resting on an inclined plain as shown in figure. Determine the reaction at all points.

\[ \text{FBD of (1)} \]

Applying eqn condition

\[ \sum F_x = 0 \]
\[ R_A - R_c \cos 80^\circ - R_B \cos 60^\circ = 0 \]
\[ R_A - 0.87 R_c - 0.5 R_B = 0 \] (1)

\[ \sum F_y = 0 \]
\[ R_B \sin 60^\circ - R_c \sin 30^\circ - W = 0 \]
\[ 0.87 R_B - 0.5 R_c = 100 \] (2)

\[ \text{FBD of (2)} \]
Two balls are resting on an inclined plane, as shown in the figure. Determine the reactions at all points.
Applying eqn condition

$$\sum f_x = 0$$

$$R_p - R_k (\cos 40^\circ) - R_q (\cos 50^\circ) = 0$$

$$R_p - R_k 0.77 - 0.64 R_q = 0 \quad \text{(1)}$$

$$\sum f_y = 0$$

$$R_q \sin 50^\circ - R_k \sin 40^\circ - w = 0$$

$$R_q 0.77 - R_k 0.64 - 98.1 = 0$$

$$0.77 R_q - 0.64 R_k = 98.1$$

FBD of (2)

$$\sum f_x = 0$$

$$R_k (\cos 40^\circ) - R_s (\cos 50^\circ) = 0$$

$$0.77 R_k - 0.64 R_s = 0$$

$$0.77 R_k = 0.64 R_s$$

$$R_k = 0.83 R_s$$
\[ \Sigma F_y = 0 \]
\[ R_k \sin 40^\circ + R_s \sin 50^\circ - 11 = 0. \]
\[ 0.83 R_s 0.64 + 0.97 R_s - 147.15 = 0. \]
\[ 1.3 R_s = 147.15. \]
\[ R_s = 113.19 \text{ N}. \]
\[ R_k = 0.83 R_s = 93.94 \text{ N}. \]  
\[ \Rightarrow \]
\[ 0.97 R_q = 98.1 + 60.12. \]
\[ R_q = 205.48 \text{ N}. \]  
\[ \Rightarrow \]
\[ R_p = 203.84 \text{ N}. \]

Two balls are resting on a container as shown in figure. Determine the reactions at all points.

\[ R = 100 \text{ mm}. \]

From \( \Delta ABC \)
\[ \Theta = \frac{150^\circ}{300^\circ} = \frac{1}{2}. \]
\[ \Theta = 60^\circ. \]
A ball of weight 1000 N thrust on a horizontal plane and is attached to two strings AB and AC which passes over pulleys as shown in figure. If string AB is horizontal, find the $\alpha$ and find reaction below ball and plane.

**FBD of ball**

$\tau_{AB}$

$\tau_{AC}$

$W$

$R$

**FBD of 400 N**

$\tau_{AC} = 400 N$

**FBN of 300 N**

$\tau_{AB}$

300 N

$300 N$
Applying eqn conditions.

\[ \sum F_x = 0 \]

\[ 400 \cos \alpha - T_{AB} = 0 \]

\[ 400 \cos \alpha = 300 \]

\[ \cos \alpha = \frac{3}{4} \]

\[ \alpha = 41.41^\circ \]

\[ \sum F_y = 0 \]

\[ R - W + T_{AC} \sin \alpha = 0 \]

\[ R - 1000 \, N + 400 \sin 41.41^\circ = 0 \]

\[ R = 735.4 \, N \]

3. A system of connected cables is supporting two loads as shown in figure. Determine the tension in various segments of the cable.
\[ \sum f_x = 0 \]
\[ T_{BC} \sin 30^\circ - T_{BD} \sin 45^\circ - T_{AB} = 0 \]

\[ \sum f_y = 0 \]
\[ T_{BC} \cos 30^\circ - T_{BD} \cos 45^\circ - W = 0 \]

Point D.

\[ \sum f_x = 0 \]
\[ T_{DE} \cos 90^\circ - T_{BD} \cos 45^\circ = 0 \]

\[ \sum f_y = 0 \]
\[ T_{DE} \sin 90^\circ + T_{BD} \sin 45^\circ - W = 0 \]
4. In the figure, \( P = 150 \text{ N} \) and \( Q = 600 \text{ N} \) are applied in a vertical plane as shown in figure. Find the tension in each string.
FBD of 150N

\[ F = 150N \]

FBD of 600N force on point A.

\[ \Sigma F_x = 0 \]

\[ T_{AC} + T_{AB}\cos 30^\circ - T_{AD} = 0 \]

\[ \Sigma F_y \]

\[ T_{AB}\sin 30^\circ - W = 0 \]
Three balls of equal size are thrusting as shown in Figure. Determine the reactions at all points and tension on the cable connecting ball 1 and 2. Radius of ball = 150 mm.

From ΔXYZ:

\[ \cos \theta = \frac{200}{500} \]
\[ \theta = \cos^{-1} \left( \frac{2}{5} \right) \]
\[ \theta = 48.19^\circ \]

\[ \theta = \sin^{-1} \left( \frac{2}{3} \right) = 46.31^\circ \]

FBD of ball 1

\[ \sum F_x = 0 \]
\[ T - R_Q \cos 48.2^\circ = 0 \]
\[ T = 0.67 R_Q \]
\[ \sum F_y = 0 \]
\[ R_p - R_Q \sin 48.2^\circ - k_1 = 0 \]
\[ R_p = R_Q \sin 48.2^\circ - k_1 = 0 \]
\[ R_p - R_Q \sin 48.2^\circ = 100 \text{ N} \quad (2) \]

400 mm.
FBD of ball 2

\[ \sum F_x = 0. \]

\[ R_R \cos 48.2 - T = 0 \]
\[ 0.67R_R - T = 0 \quad (3) \]

\[ \sum F_y = 0. \]

\[ R_s - k_l - R_R \sin 48.2 = 0 \]
\[ R_s - 0.74R_R = 100 \quad (4) \]

FBD of ball 3

\[ \sum F_x = 0. \]

\[ R_Q \sin 41.8 - R_R \sin 41.8 = 0. \quad (5) \]
\[ R_Q = R_R. \]

\[ \sum F_y = 0. \]

\[ R_Q \cos 41.8 + R_R \cos 41.8 - k_l = 0. \quad (6) \]

(3) + (2)

\[ R_Q \cos 41.8 + R_R \cos 41.8 = 100. \]

\[ R_R = 41.8 \]
\[ R_Q = 67.07, \quad N = R_R. \]
(4) \[ R_3 = 149.63 \text{ N} \]

(3) \[ T = 44.93 \text{ N} \]

(2) \[ R_D = 150.30 \text{ N} \]

\[ R_D = 67.07 \text{ N} \]

\[ R_R = 67.07 \text{ N} \]

Two balls of diameter 500 mm are resting as shown in Fig. Determine the reaction at all points.

FBD of ball 1

\[ \Sigma F_x = 0 \]

\[ R_A + R_B \sin 40^\circ - R_D \cos 40^\circ = 0 \quad (1) \]

\[ \Sigma F_y = 0 \]

\[ R_B - W > 0 \quad \rightarrow \quad (2) \]

\[ R_B = 200 \text{ N} \]
\[ R_B \cos 50^\circ - R_C \sin 40^\circ - W = 0 \quad (2) \]

\[ \Sigma F_x = 0 \]

\[ R_c \cos 40^\circ - R_d \cos 50^\circ = 0 \]

\[ R_c = R_d 0.84. \quad (1) \]

\[ \Sigma F_y = 0 \]

\[ R_c \sin 40^\circ + R_d \sin 50^\circ - k_1 = 0 \quad (2) \]

\[ 0.84 R_d \sin 40^\circ + R_d \sin 50^\circ = 200 \]

\[ R_d = 158.14\, \text{N} \]

\[ R_c = 128.63\, \text{N} \]

\[ R_B = 439.77\, \text{N} \]

\[ R_A = 465.2 \]

Two loads are hanging on a table as shown in Fig. Determine the tension in each segment of the string.
\[ \sum F_x = 0. \]
\[ T_{EF} \cos 50^\circ - T_{DE} \cos 20^\circ - T_{DE} = 0 \quad \text{(1)} \]

\[ \sum F_y = 0. \]
\[ T_{EF} \sin 50^\circ + T_{CE} \sin 20^\circ - W = 0. \]
\[ T_{EF} \sin 50^\circ + T_{CE} \sin 20^\circ = 200 \quad \text{(2)} \]

\[ \sum F_x = 0. \]
\[ T_{DE} - T_{BD} \cos 30^\circ = 0. \quad \text{(3)} \]

\[ \sum F_y = 0. \]
\[ T_{BD} \sin 30^\circ - W = 0. \quad \text{(4)} \]
\[ T_{BD} = \frac{200}{\sin 30^\circ} = 600 \text{ N} \]
A ball of radius 40 cm is moving over a rectangular block height 20 cm as shown in Fig. 1. A force P is applied on the ball through a string passing over the circumference of the ball. Assume ground is smooth.

i) Minimum force P required to be applied on string

ii) Reaction at points A + B

---

For a system of three forces to maintain the body in eqm all three forces must pass through a single point, i.e., point D.

From \( \Delta B E C \)

\[ CE = 40 - 20 = 20 \text{ cm} \]

\[ BE = \sqrt{BE^2 - CE^2} = \sqrt{40^2 - 20^2} = \sqrt{1200} = 34.64 \text{ cm} \]

Consider \( \Delta BEO \).

\[ EP = 40 + 20 = 60 \text{ cm} \]

\[ BE = 34.64 \text{ cm} \]

\[ \theta = \tan^{-1} \left( \frac{34.64}{60} \right) = 29.99^\circ \approx 30^\circ \]

FBD

\[ RA = 0 \text{ (smooth surface)} \]
\[
\sum f_x = 0.
\]

\[
P - R_B \sin 30^\circ = 0. \quad (1)
\]

\[
\sum f_y = 0.
\]

\[
R_B \cos 30^\circ - W = 0 \quad (2)
\]

\[
R_B = \frac{1000}{\cos 30^\circ} = 1154.7 \text{ N.}
\]

\[
P = \frac{577.35}{14000} \text{ N.}
\]